

□ 24 □ □□□□□□□□□

1□□□□□ $f(x) = x^3 + ax + \frac{1}{4}$ □ $g(x) = -\ln x$ □

□1□□ a □□□□□ x □□□□ $y = f(x)$ □□□□

□2□□ $F(x) = f(x) - g(x)$ □ $[1, +\infty)$ □□□□□□ a □□□□□□

□3□□ $\min\{m, n\}$ □ □□ m, n □□□□□□□□□□ $h(x) = \min\{f(x), g(x)\} (x > 0)$ □□□ $h(x)$ □□□□□□

□□□□□□□1□ $f(x) = 3x^2 + a$ □

□□□ $y = f(x)$ □ x □□□□□ $F(x_0, 0)$ □

□ $f(x_0) = 0$ □ $f'(x_0) = 0$ □

$$\begin{cases} x_0^3 + ax_0 + \frac{1}{4} = 0 \\ 3x_0^2 + a = 0 \end{cases}$$
 □

□□ $x_0 = \frac{1}{2}$ □ $a = -\frac{3}{4}$ □

□□□ $a = -\frac{3}{4}$ □□ x □□□□□ $y = f(x)$ □□□□

□2□ $F(x) = f(x) - g(x) = x^3 + ax + \frac{1}{4} + \ln x$ □

□□□ $F(x) = 3x^2 + a + \frac{1}{x}$ □

□□□□□ $3x^2 + a + \frac{1}{x} \dots 0$ □ $[1, +\infty)$ □□□□

□□ $-a, 3x^2 + \frac{1}{x}$ □□□□□

□ $3x^2 + \frac{1}{x}$ □□□□□ $6x - \frac{1}{x^2} > 0$ □ $x \geq 1$ □□□

□□□□□□ 4□

□ $-a, 4$ □□□ $a \dots -4$ □

$$\square \square \square x \in (1, +\infty) \square \square g(x) = -\ln x < 0 \square$$

$$\therefore \square \square h(x) = \min \{ f(x) \square g(x) \}, \square g(x) < 0 \square$$

$$\square h(x) \square x \in (1, +\infty) \square \square \square \square \square$$

$$\square x=1 \square \square \square a > -\frac{5}{4} \square \square f \square 1 \square = a + \frac{5}{4} > 0 \square$$

$$\therefore h(x) = \min \{ f \square 1 \square \square g \square 1 \square \} = g \square 1 \square = 0 \square$$

$$\square x=1 \square \square \square h(x) \square \square \square \square \square \square$$

$$\square a < -\frac{5}{4} \square \square f \square 1 \square = a + \frac{5}{4} < 0 \square$$

$$\therefore h(x) = \min \{ f \square 1 \square \square g \square 1 \square \} = f \square 1 \square < 0 \square$$

$$\square x=1 \square \square \square \square h(x) \square \square \square \square$$

$$\square x \in (0, 1) \square \square g(x) = -\ln x > 0 \square$$

$$\square \square \square \square f(x) \square (0, 1) \square \square \square \square \square \square \square \square$$

$$\textcircled{1} \square a, -3 \square a, 0 \square \square f(x) = 3x^2 + a \square (0, 1) \square \square \square \square \square$$

$$\square \square f(x) \square \square \square (0, 1) \square \square \square \square$$

$$\square f(0) = \frac{1}{4} \square \square f \square 1 \square = a + \frac{5}{4} \square$$

$$\therefore \square a, -3 \square \square \square \square f(x) \square \square \square (0, 1) \square \square \square \square \square \square \square$$

$$\square a, 0 \square \square \square \square f(x) \square \square \square (0, 1) \square \square \square \square \square \square$$

$$\textcircled{2} \square -3 < a < 0 \square \square \square \square f(x) \square (0, \sqrt{\frac{-a}{3}}) \square \square \square \square \square \square (\sqrt{\frac{-a}{3}} \square 1) \square \square \square \square \square \square$$

$$\square\square \quad x=\sqrt{\frac{-a}{3}} \quad \square\square \quad f(x) \quad \square\square\square\square\square \quad f(\sqrt{\frac{-a}{3}})=\frac{2a}{3}\sqrt{\frac{-a}{3}}+\frac{1}{4} \quad \square$$

$$\square \quad f(\sqrt{\frac{-a}{3}})>0 \quad \square\square \quad -\frac{3}{4}<a<0 \quad \square\square \quad f(x) \quad \square \quad (0,1) \quad \square\square\square\square\square$$

$$\square \quad f(\sqrt{\frac{-a}{3}})=0 \quad \square\square \quad a=-\frac{3}{4} \quad \square\square \quad f(x) \quad \square \quad (0,1) \quad \square\square\square\square\square\square\square$$

$$\square \quad f(\sqrt{\frac{-a}{3}})<0 \quad \square\square \quad -3<a<-\frac{3}{4} \quad \square\square \quad f(0)=\frac{1}{4} \quad \square \quad f \quad \square 1 \quad \square =a+\frac{5}{4} \quad \square$$

$$\square \quad \square \quad -\frac{5}{4}<a<-\frac{3}{4} \quad \square\square \quad f(x) \quad \square \quad (0,1) \quad \square\square\square\square\square\square\square$$

$$\square \quad -3<a, -\frac{5}{4} \quad \square\square \quad f(x) \quad \square \quad (0,1) \quad \square\square\square\square\square\square\square$$

$$\square\square\square\square\square \quad a>-\frac{3}{4} \quad a<-\frac{5}{4} \quad \square\square \quad h(x) \quad \square\square\square\square\square\square$$

$$\square \quad a=-\frac{3}{4} \quad -\frac{5}{4} \quad \square\square \quad h(x) \quad \square\square\square\square\square\square$$

$$\square \quad -\frac{5}{4}<a<-\frac{3}{4} \quad \square\square\square \quad h(x) \quad \square\square\square\square\square\square$$

$$2\square\square\square\square\square \quad f(x)=x^2+ax+\frac{1}{4} \quad \square \quad g(x)=-\ln x \quad \square$$

$$\square 1\square\square\square\square \quad g[f(x)] \quad \square\square\square\square \quad R \quad \square\square\square\square \quad a \quad \square\square\square\square\square\square$$

$$\square 2\square\square\square\square \quad g[f(x)] \quad \square \quad (1,+\infty) \quad \square\square\square\square\square\square\square\square \quad a \quad \square\square\square\square\square\square$$

$$\square 3\square\square \quad \min\{m \quad n\} \quad \square\square \quad m \quad n \quad \square\square\square\square\square\square\square\square \quad h(x)=\min\{f(x) \quad g(x)\} \quad (x>0) \quad \square\square\square \quad h(x) \quad \square\square\square\square\square\square$$

$$\square\square\square\square\square\square\square 1\square\square\square\square \quad g[f(x)] \quad \square\square\square\square \quad R \quad \square$$

$$\square\square\square \quad x \in R \quad \square\square\square \quad f(x)=x^2+ax+\frac{1}{4}>0 \quad \square$$

$$\square\square \triangle \quad =a^2-4 \times 1 \times \frac{1}{4}<0 \quad \square\square\square \quad -1<a<1 \quad \square$$

$$\text{证明 } a \text{ 是 } (-1, 1) \text{ 的根}$$

$$\text{证明 } [f(x)]_{(1, +\infty)} \text{ 有界}$$

$$\text{证明 } g(x) \text{ 在 } (0, +\infty) \text{ 有界}$$

$$\text{证明 } f(x) \text{ 在 } (1, +\infty) \text{ 有界} \quad x \in (1, +\infty) \quad f(x) > 0$$

$$\text{证明 } -\frac{a}{2} - 1 \quad f'(x) > 0$$

$$\text{证明 } -\frac{a}{2} - 1 \quad 1 + a + \frac{1}{4} > 0$$

$$\text{证明 } a > -\frac{5}{4}$$

$$\text{证明 } a \text{ 是 } (-\frac{5}{4}, +\infty) \text{ 的根}$$

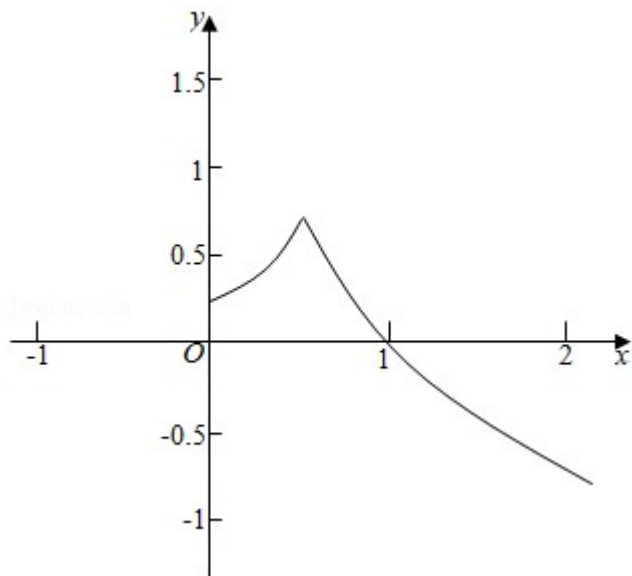
$$\text{证明 } x > 1 \quad g(x) = -\ln x < 0$$

$$\text{证明 } h(x) = \min\{f(x), g(x)\}, \quad g(x) < 0$$

$$\text{证明 } h(x) \text{ 在 } (1, +\infty) \text{ 有界}$$

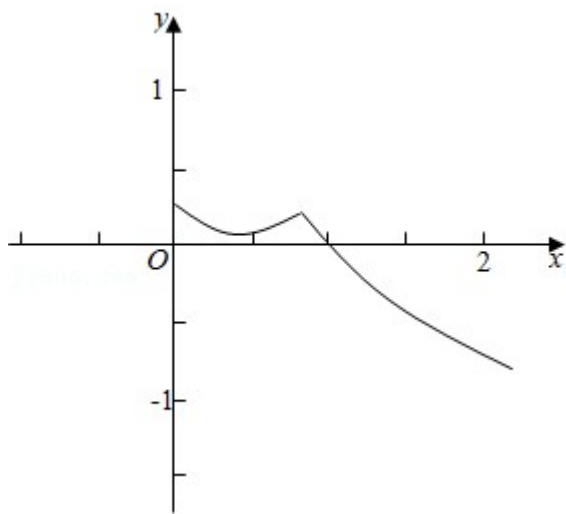
$$\text{① 证明 } a \neq 0 \quad f(x) \text{ 在 } (0, \frac{1}{4}) \text{ 有界} \quad -\frac{a}{2} - 0$$

$$\text{证明 } h(x) \text{ 有界} \quad h(x) \text{ 有界} \quad x=1$$

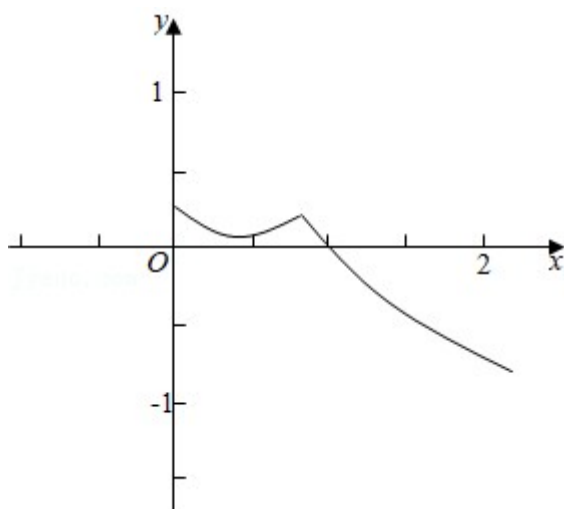


② $a < 0$ $f(x)$ $(0, \frac{1}{4})$ $-\frac{a}{2} > 0$

$= a^2 - 4 \times 1 \times \frac{1}{4} < 0$
 Δ $-1 < a < 0$ $h(x)$ $x=1$

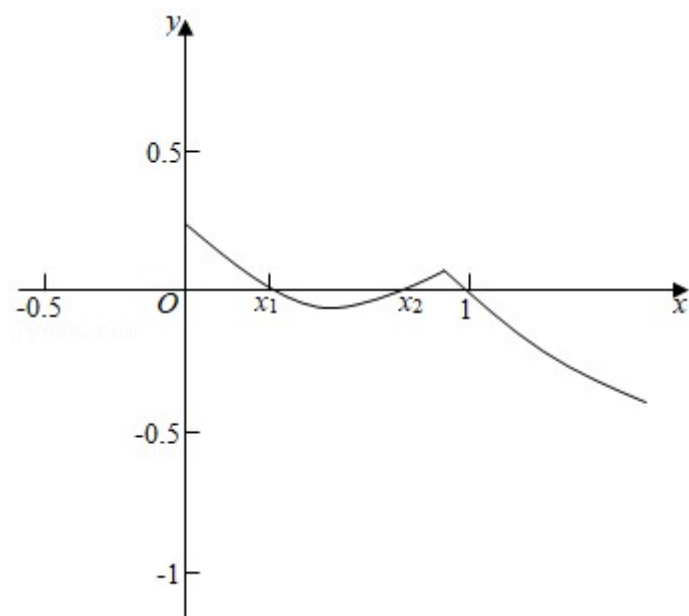


$= a^2 - 4 \times 1 \times \frac{1}{4} = 0$ $a = -1$ $f(x)$ $x = -\frac{a}{2} = \frac{1}{2}$ $h(x)$ $x = \frac{1}{2}$ $x=1$

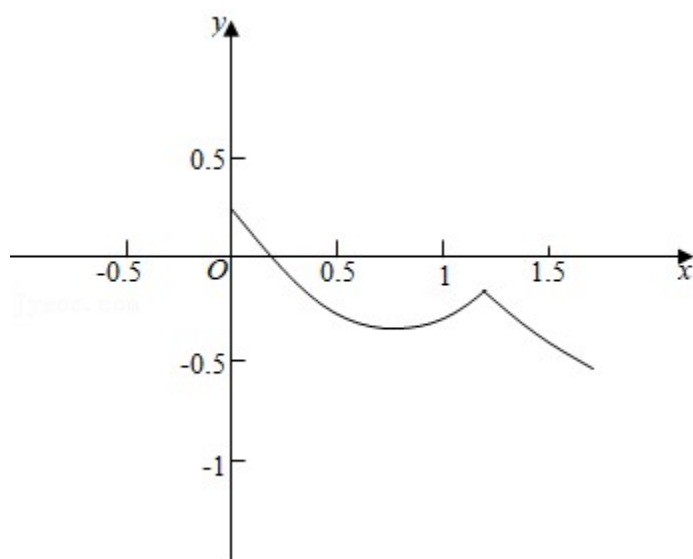


$$\Delta = a^2 - 4 \times 1 \times \frac{1}{4} > 0 \quad a < -1 \quad f(x) = 0 \quad x_1 = \frac{-a - \sqrt{a^2 - 1}}{2} \quad x_2 = \frac{-a + \sqrt{a^2 - 1}}{2} \quad 0 < x_1 < 1 \quad 0 < x_2$$

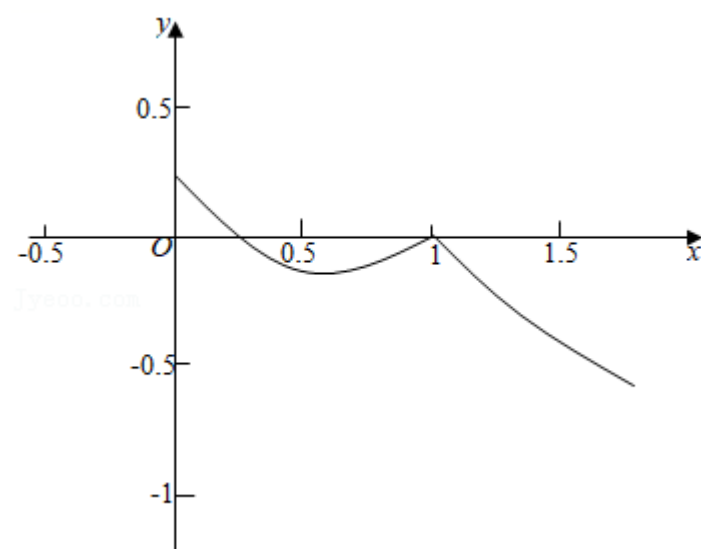
$$x_2 = \frac{-a + \sqrt{a^2 - 1}}{2} < 1 \quad -\frac{5}{4} < a < -1 \quad h(x) \quad 3 \quad x = x_1 \quad x = x_2 \quad x = 1$$



$$x_2 = \frac{-a + \sqrt{a^2 - 1}}{2} > 1 \quad a < -\frac{5}{4} \quad h(x) \quad 1 \quad x = x_1$$



$x_2 = \frac{-a + \sqrt{a^2 - 1}}{2} = 1$
 $a = -\frac{5}{4}$
 $f(x)$
 2
 $x = x_1$
 $x = 1$



$a \in (-\infty, -\frac{5}{4}) \cup (-1, 0)$
 $f(x)$

$a = -1$
 $-\frac{5}{4}$
 $f(x)$

$a \in (-\frac{5}{4}, -1)$
 $f(x)$

$f(x) = x^3 - 3ax + \epsilon$
 $g(x) = 1 - \ln x$
 ϵ

$f(x)$

2. $\max\{m, n\} = m, n$ \Rightarrow $\max\{f(x), g(x)\} = f(x)$ ($x > 0$) \Rightarrow $\max\{f(x), g(x)\} = f(x)$ ($0, +\infty$) \Rightarrow 2. a

3. $a > 0$

$$f(x) = 3x^2 - 3a$$

$$a, 0 \Rightarrow f(x) \dots 0 \Rightarrow f(x) \in R$$

$$a > 0 \Rightarrow f(x) = 3(x + \sqrt{a})(x - \sqrt{a})$$

$$x \in (-\infty, -\sqrt{a}) \cup (\sqrt{a}, +\infty) \Rightarrow f(x) > 0 \Rightarrow f(x) \dots$$

$$x \in (-\sqrt{a}, \sqrt{a}) \Rightarrow f(x) < 0 \Rightarrow f(x) \dots$$

$$2. x \in (0, e) \Rightarrow g(x) > 0 \Rightarrow h(x) \dots g(x) > 0 \Rightarrow h(x) \in (0, e) \dots$$

$$x = e \Rightarrow g(e) = 0 \Rightarrow f(e) = e^2 - 3ae + e$$

$$f(e) \dots 0 \Rightarrow a \dots \frac{e^2 + 1}{3} \Rightarrow e \dots h(x) \dots$$

$$f(e) > 0 \Rightarrow a < \frac{e^2 + 1}{3} \Rightarrow e \dots h(x) \dots$$

$$x \in (e, +\infty) \Rightarrow g(x) < 0 \Rightarrow f(x) \dots f(x) = 3x^2 - 3a > 3e^2 - 3a$$

$$1. a, e^2 \Rightarrow f(x) > 0 \Rightarrow f(x) \in (e, +\infty) \dots$$

$$a, \frac{e^2 + 1}{3} \Rightarrow f(e) \dots 0 \Rightarrow f(x) \in (e, +\infty) \dots$$

$$\frac{e^2 + 1}{3} < a, e^2 \Rightarrow f(e) < 0 \Rightarrow f(2e) = 8e^2 - 6ae + e \dots 8e^2 - 6e^2 + e > 0 \Rightarrow f(x) \in (e, +\infty) \dots$$

$$2. a > e^2 \Rightarrow f(x) \in (e\sqrt{a}, \dots (\sqrt{a}, +\infty) \dots$$

$$\text{□□□ } f'_{\text{□e□}} = e^x - 3ae + e < e^x - 3e^x + e < 0 \quad \text{□ } f(2a) = 8a^3 - 6a^2 + e > 8a^2 - 6a^2 + e = 2a^2 + e > 0 \quad \text{□□□□□ } f(x) \quad \text{□□□□}$$

□□□

$$\text{□□□ } a > \frac{e^2 + 1}{3} \quad \text{□}$$

$$4 \text{□□□□□ } f(x) = \ln x - x^2 + ax \quad \text{□ } g(x) = e^x - e \quad \text{□□□□ } a > 0 \quad \text{□}$$

$$\text{□!□□□□□ } \ln x, x < 1 \quad \text{□}$$

$$\text{□□□□ } a = 2 \text{□□□□ } f(x) < \frac{5}{4} \quad \text{□}$$

$$\text{□□□□□ } \max\{m_1, n_1\} \text{□□ } m_1, n_1 \text{□□□□□□□□□□□ } h(x) = \max\{f(x), g(x)\} \text{□□□□□□ } h(x) \text{□ } (0, +\infty) \text{□□□□□□□□□□}$$

$$\text{□□□□□□□□□□□□□ } \varphi(x) = \ln x - x + 1 \quad \text{□□ } \varphi'(x) = \frac{1}{x} - 1, x > 0 \quad \text{□}$$

$$\text{□ } \varphi'(x) = 0 \text{□ } x = 1 \text{□□□□ } (0, 1) \text{□□ } \varphi'(x) > 0 \text{□ } \varphi(x) \text{□□□□ } (1, +\infty) \text{□□ } \varphi'(x) < 0 \text{□ } \varphi(x) \text{□□□□}$$

$$\text{□□ } \varphi(x), \varphi \text{□1□} = 0 \text{□□ } \ln x, x < 1 \quad \text{□}$$

$$\text{□□□□□□□□ } a = 2 \text{□□ } f(x) = \ln x - x^2 + 2x, x < 1 - x^2 + 2x = -\left(x - \frac{3}{2}\right)^2 + \frac{5}{4}, \frac{5}{4} \quad \text{□}$$

$$\text{□□□□ } „ \text{ ”□□ } x = 1 \text{□□□□□□□□ } „ \text{ ”□□ } x = \frac{3}{2} \text{□□□□□□□□□□□□□□}$$

$$\text{□□ } f(x) < \frac{5}{4} \quad \text{□}$$

$$\text{□□□□□□□□□□ } (1, +\infty) \text{□□ } g(x) > 0 \text{□□□□ } h(x) = \max\{f(x), g(x)\}, g(x) > 0 \quad \text{□}$$

$$\text{□□ } h(x) = \max\{f(x), g(x)\}, g(x) > 0 \text{□□□□ } (1, +\infty) \text{□□□□□□□□□□}$$

$$\text{□□□□□□□□□□ } (0, 1) \text{□□ } x = 1 \text{□□□□□□}$$

$$\text{□□□□ } f(x) \text{□□□□□□□□ } (0, +\infty) \text{□ } f(x) = \frac{1}{x} - 2x + a = \frac{-2x^2 + ax + 1}{x} \quad \text{□}$$

$$\square f(x_0)=0 \quad \square x_0=\frac{a+\sqrt{a^2+8}}{4} \quad \square \square \square \square \square \square$$

$$\square (0, x_0) \quad \square f(x) > 0 \quad \square f(x) \quad \square \square \square \square (x_0 + \infty) \quad \square f(x) < 0 \quad \square f(x) \quad \square \square \square f(x)_{\max} = f(x_0) \quad \square$$

$$\textcircled{1} \quad \square a=1 \quad \square x_0=1 \quad \square \square \square \quad f(x)_{\max} = f \quad \square 1 \square = 0 \quad \square$$

$$\square \square \square \square (0,1) \quad \square g(x) < 0 \quad \square g \quad \square 1 \square = 0 \quad \square \square \square \square h(x) \quad \square \square \square \square \square \square x=1 \quad \square$$

$$\textcircled{2} \quad \square 0 < a < 1 \quad \square x_0 = \frac{a + \sqrt{a^2 + 8}}{4} < 1 \quad \square \square \square \quad f(x_0) = \frac{1}{x_0} - 2x_0 + a = 0 \quad \square \square \square \quad a = 2x_0 - \frac{1}{x_0} \quad \square$$

$$\square \square \quad f(x_0) = \ln x_0 - x_0^2 + x_0(2x_0 - \frac{1}{x_0}) = \ln x_0 + x_0^2 - 1 < \ln 1 + 1^2 - 1 = 0 \quad \square$$

$$\square \square \quad f(x) < 0 \quad \square \square \square \square$$

$$\square \square \square \quad g(x) \quad \square \square \square \square \square \square h(x) \quad \square \square \square \square \square \square x=1 \quad \square$$

$$\textcircled{3} \quad \square a > 1 \quad \square x_0 = \frac{a + \sqrt{a^2 + 8}}{4} > 1 \quad \square \square \square \quad f(x) \quad \square (0,1) \quad \square \square \square \square$$

$$\square \square \square \quad f \quad \square 1 \square = a - 1 > 0 \quad \square h(\frac{1}{2a}) = n \frac{1}{2a} - \frac{1}{4a^2} + \frac{1}{2} < \frac{1}{2a} - 1 - \frac{1}{4a^2} + \frac{1}{2} = -(\frac{1}{2a} - \frac{1}{2})^2 - \frac{1}{4} < 0 \quad \square$$

$$\square \square \quad f(x) \quad \square \square \square \quad (0,1) \quad \square \square \square \square \square \square x=x_1 \quad \square$$

$$\square \square \square \quad g(x) \quad \square \square \square \square \square \quad x=x_1 \quad \square h(x) \quad \square \square \square \square \square$$

$$\square \square \square \square \quad 0 < a, 1 \quad \square \square \quad h(x) \quad \square (0, +\infty) \quad \square \square \square \square \square \square x=1 \quad \square$$

$$\square \quad a > 1 \quad \square \square \quad h(x) \quad \square (0, +\infty) \quad \square \square \square \quad 1 \quad \square \square \square \square$$

$$5 \quad \square \square \square \square \square \quad f(x) = (x-4)e^{x-3} - \frac{1}{2}x^2 + 3x - \frac{7}{2} \quad \square g(x) = ae^x + \cos x \quad \square \square \square \quad a \in R \quad \square$$

$$\square 1 \quad \square \square \square \square \square \quad f(x) \quad \square \square \square \square \square \square \square \square \square \quad f(x) > 0 \quad \square \square \square \square$$

2. $a=1$ $x>0$ $g(x)>2$

3. $\max\{m, n\}$ m, n $h(x)=\max\{f(x), g(x)\}$ $h(x)..0$ $(0, +\infty)$ a

1. $f(x)=(x-3)e^{x-3}-x+3=(x-3)(e^{x-3}-1)$ 1

$x>3$ $x-3>0$ $e^{x-3}-1>0$ $\therefore f(x)>0$

$x<3$ $x-3<0$ $e^{x-3}-1<0$ $\therefore f(x)>0$

$x=3$ $f(x)=0$ 2

$x \in R$ $f(x)..0$ $f(x)$ R 3

$f(3)=0$ $f(x)>0$ $(3, +\infty)$ 4

2. $g(x)=e^x-\sin x$ 5

$x>0$ $e^x>1$ $\sin x \in [-1, 1]$ 6

$g(x)=e^x-\sin x>0$ $g(x)$ $(0, +\infty)$ 7

$g(x)>g(0)=2$ $g(x)>2$ 8

3. 1

$x, 3$ $f(x)..0$ $h(x)..0$

$x<3$ $f(x)<0$ $h(x)=\max\{f(x), g(x)\}$ $h(x)..0$

$g(x)..0$ $(0, 3)$ 9

$g(x)=ae^x+\cos x..0$ $a..-\frac{\cos x}{e^x}$

$h(x)=-\frac{\cos x}{e^x}$ $x \in [0, 3]$

□ $\phi(x)$ □ $(0,1)$ □ □ □ □ □ □ $(1,+\infty)$ □ □ □ □ □ □ □3 □□

□□ $\phi(x) \dots \phi(1) = 0$ □□ $f(x) \dots 0$ □ □ □ □ □ □ □4 □□

□2 □ □ □ □ □ □ $h(x) = 1 - \ln x$ □

① □ $0 < x < e$ □□ $\varphi(x) \dots h(x) > 0$ □ □ □ □ □ □ □ □ □5 □□

② □ $x = e$ □□ $h(e) = 0$ □ $g(e) = e^3 - 3ae + e$

a □□ $g(e) = e^3 - 3ae + e, 0$ □□ $a < \frac{e^2 + 1}{3}$ □□ $x = e$ □□ $\varphi(x)$ □ □ □ □ □ □

b □□ $g(e) = e^3 - 3ae + e > 0$ □□ $a < \frac{e^2 + 1}{3}$ □□ $x = e$ □□ $\varphi(x)$ □ □ □ □ □ □ □6 □□

③ □ $x > e$ □□ $h(x) < 0$ □ □ □ □ □ □ □ □ □ □ $g(x)$ □ $(e, +\infty)$ □ □ □ □ □ □

□ $g'(x) = 3x^2 - 3a$

a □□ a, e^2 □□ $g'(x) > 0$ □ $g(x)$ □ $(e, +\infty)$ □ □ □ □ □ □ □ □ $g(e) = e^3 - 3ae + e$ □

□ $a < \frac{e^2 + 1}{3}$ □□ $g(e) > 0$ □□ $g(x)$ □ $(e, +\infty)$ □ □ □ □ □ □ $\varphi(x)$ □ $(0, +\infty)$ □ □ □ □ □ □

□ $a = \frac{e^2 + 1}{3}$ □□ $g(e) = 0$ □□ $g(x)$ □ $(e, +\infty)$ □ □ □ □ □ □ $\varphi(x)$ □ $(0, +\infty)$ □ □ 1 □ □ □ □ □

□ $\frac{e^2 + 1}{3} < a, e^2$ □ □ □ □ $g(e) < 0$ □ $g(2e) = 8e^3 - 6ae + e, 8e^3 - 6e^2 + e > 0$ □□ $g(x)$ □ $(e, +\infty)$ □ □ □ □ □ □ □ □ $\varphi(x)$ □

$(0, +\infty)$ □ □ 2 □ □ □ □ □

□□ $\frac{e^2 + 1}{3} < a, e^2$ □ □ □9 □□

$$b \begin{cases} a > e^2 \end{cases} \begin{cases} g'(x) = 0 \end{cases} \begin{cases} x = \pm \sqrt{a} \end{cases}$$

$$\begin{cases} g'(x) < 0 \end{cases} \begin{cases} e < x < \sqrt{a} \end{cases} \begin{cases} g'(x) > 0 \end{cases} \begin{cases} x > \sqrt{a} \end{cases} \begin{cases} g'(x) < 0 \end{cases}$$

$$\begin{cases} g(x) \end{cases} \begin{cases} (e, \sqrt{a}) \end{cases} \begin{cases} g(x) \end{cases} \begin{cases} (\sqrt{a}, +\infty) \end{cases}$$

$$\begin{cases} g'(e) < 0 \end{cases} \begin{cases} g(2a) = 8a^3 - 6a^2 + e \cdot 2a^2 + e > 0 \end{cases} \begin{cases} g(x) \end{cases} \begin{cases} (e, +\infty) \end{cases} \begin{cases} g'(x) \end{cases} \begin{cases} (0, +\infty) \end{cases} \begin{cases} 2 \end{cases}$$

$$\begin{cases} a > e^2 \end{cases} \begin{cases} \end{cases} \dots \begin{cases} 11 \end{cases}$$

$$\begin{cases} a > \frac{e^2 + 1}{3} \end{cases} \begin{cases} g(x) \end{cases} \begin{cases} (0, +\infty) \end{cases} \begin{cases} \end{cases} \dots \begin{cases} 12 \end{cases}$$

$$7 \begin{cases} f(x) = \frac{2}{3}x^3 - 2x^2 + \frac{4}{3} \end{cases} \begin{cases} g(x) = e^x - a \forall x \in \mathbb{R} \end{cases}$$

$$\begin{cases} 1 \end{cases} \begin{cases} f(x) \end{cases} \begin{cases} [a-5, a-1] \end{cases} \begin{cases} \frac{4}{3} \end{cases} \begin{cases} a \end{cases}$$

$$\begin{cases} 2 \end{cases} \begin{cases} h(x) = \frac{3}{2}f(x) - x + 1 \end{cases} \begin{cases} F(x) = \begin{cases} h(x), h(x), g(x) \\ g(x), h(x) > g(x) \end{cases} \end{cases} \begin{cases} x_1, x_2, \dots, x_n \end{cases} \begin{cases} F(x) \end{cases} \begin{cases} a, e^3 \end{cases} \begin{cases} F(x) \end{cases}$$

$$\begin{cases} 1 \end{cases} \begin{cases} f(x) = 2x^2 - 4x = 2x(x-2) \end{cases}$$

$$\therefore f(x) \begin{cases} (-\infty, 0) \end{cases} \begin{cases} (2, +\infty) \end{cases} \begin{cases} (0, 2) \end{cases}$$

$$\begin{cases} f(x) \end{cases} \begin{cases} f(0) = \frac{4}{3} \end{cases} \begin{cases} f(x) \end{cases} \begin{cases} f(2) = -\frac{4}{3} \end{cases}$$

$$\begin{cases} f(3) = \frac{4}{3} \end{cases} \therefore \begin{cases} f(x) \end{cases} \begin{cases} [a-5, a-1] \end{cases} \begin{cases} \frac{4}{3} \end{cases}$$

$$\begin{cases} \begin{cases} a-5, 0 \\ 0, a-1, 3 \end{cases} \end{cases} \begin{cases} 1, a, 4 \end{cases}$$

$$\begin{cases} 2 \end{cases} \begin{cases} h(x) = \frac{3}{2}f(x) - x + 1 = x^3 - 3x^2 - x + 3 = (x+1)(x-1)(x-3) \end{cases}$$

$$\begin{cases} x, -1 \end{cases} \begin{cases} g(x) = e^x - ax > 0 \end{cases} \begin{cases} F(x) = h(x) \end{cases}$$

$$\therefore F(x) \text{ 在 } (-\infty, -1] \text{ 上单调递减 } x_1 = -1$$

$$\text{当 } x > -1 \text{ 时 } g'(x) = e^x - a \therefore g'(x) \text{ 在 } (0, \ln a) \text{ 上单调递减 } (\ln a, +\infty) \text{ 上单调递增}$$

$$\text{当 } a = e^3 \text{ 时 } \ln a = 3$$

$$\text{当 } g(0) = 1 > 0 \text{ 时 } g(1) = e - a < 0 \text{ 当 } x \in (-1, 1) \text{ 时 } f(x) > 0$$

$$\therefore F(x) \text{ 在 } (0, 1) \text{ 上单调递减 } x_2$$

$$\text{当 } g(\ln a) = a(1 - \ln a) < 0$$

$$\text{当 } k(x) = x - \ln x(x \cdot e^3) \text{ 时 } k'(x) = \frac{x-1}{x} > 0$$

$$\therefore k(x) \text{ 在 } [e^3, +\infty) \text{ 上单调递增 } k(x) = x - \ln x, k(e^3) = e^3 - 3 > 0$$

$$\therefore a > \ln a \text{ 时 } g(a) = e^a - a^2$$

$$\text{当 } \varphi(x) = e^x - x^2(x, 2) \text{ 时 } \varphi' = e^x - 2x, \varphi'(x) = e^x - 2 > 0$$

$$\therefore \varphi' \text{ 在 } [2, +\infty) \text{ 上单调递增 } \varphi'(x) > \varphi'(2) = e^2 - 4 > 0$$

$$\therefore \varphi(x) \text{ 在 } [2, +\infty) \text{ 上单调递增 } \varphi(x) > \varphi(2) = e^2 - 4 > 0 \text{ 当 } g(a) > 0$$

$$\therefore F(x) \text{ 在 } (\ln a, a) \text{ 上单调递减 } x_3$$

$$\text{当 } a = e^3 \text{ 时 } F(x) \text{ 在 } x = -1, 0 < x_2 < 1, \ln a < x_3 < a$$

$$\text{当 } x_1 < x_2 < x_3$$

$$8 \text{ 个 } f(x) = -x^2 + \frac{1}{2}x^2 + mx$$

$$\text{1 个 } m=2 \text{ 个 } f(x) \text{ 个}$$

$$\text{2 个 } g(x) = mx \{ f(x) - \frac{1}{2}x^2 - \frac{1}{4}(e^x - e) \} \text{ 个 } mx \text{ 个 } e \text{ 个 } m, \frac{3}{4} \text{ 个 } g(x)$$

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$$\square \square \square \square \square \square 1 \square \quad m=2 \square \square \quad f(x) = -x^2 + \frac{1}{2}x^2 + 2x \quad \square$$

$$f'(x) = -(x-1)(3x+2) \quad \square$$

$$\square \quad f'(x) > 0 \quad \square \square \square \square \quad -\frac{2}{3} < x < 1 \quad \square$$

$$\square \quad f'(x) < 0 \quad \square \square \square \square \quad x > 1 \quad \square \quad x < -\frac{2}{3} \quad \square$$

$$\square \quad f(x) \quad \square \quad (-\infty, -\frac{2}{3}) \quad \square \square \square \square \quad (-\frac{2}{3}, 1) \quad \square \square \square \square \quad (1, +\infty) \quad \square \square \square$$

$$\square \quad f(x)_{\square \square \square} = f_{\square 1 \square} = \frac{3}{2} \quad \square$$

$$\square 2 \square \square \square \quad h(x) = e^x - e \quad \square \quad R \square \square \square \square \square \square \quad x=1 \quad \square \square \quad 1 \quad \square \square \square \square \square \quad x < 1 \quad \square \square \quad h(x) < 0 \quad \square$$

$$\square \quad F(x) = f(x) - \frac{1}{2}x^2 - \frac{1}{4} = -x^2 + mx - \frac{1}{4} \quad \square$$

$$F(x) = -3x^2 + m \quad \square$$

$$\textcircled{1} \quad m, 0 \quad \square \square \quad F(x), 0 \quad \square \quad F(x) \quad \square \quad R \square \square \square \square \square \quad (0, -\frac{1}{4}) \quad \square \quad F(-1) = \frac{3}{4} - m > 0 \quad \square$$

$$\square \quad F(x) \quad \square \quad x, 0 \quad \square \square \square \quad 1 \quad \square \square \square \square \square \square \quad y = g(x) \quad \square \quad 2 \quad \square \square \square \square$$

$$\textcircled{2} \quad m > 0 \quad \square \square \square \quad F(x) = 0 \quad \square \square \square \square \square \quad x_1 = -\sqrt{\frac{m}{3}} < 0 \quad \square \quad x_2 = \sqrt{\frac{m}{3}} > 0 \quad \square$$

$$\square \quad -\sqrt{\frac{m}{3}} \quad \square \square \square \quad F(x) \quad \square \square \square \square \square \square \quad \sqrt{\frac{m}{3}} \quad \square \quad F(x) \quad \square \square \square \square \square \square$$

$$F(-\sqrt{\frac{m}{3}}) = -\frac{2m}{3}\sqrt{\frac{m}{3}} - \frac{1}{4} < 0 \quad \square$$

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$$F(\sqrt{\frac{m}{3}}) = \frac{2m}{3}\sqrt{\frac{m}{3}} - \frac{1}{4} \quad \square$$

$$\square \quad F(\sqrt{\frac{m}{3}}) < 0 \quad \square \quad m < \frac{3}{4} \quad \square \square \square \square \quad F(x) \square (0, +\infty) \quad \square \square \square \square \quad 0 \square \square \square \square \quad y = g(x) \quad \square \quad 2 \quad \square \square \square \square$$

$$\square \quad F(\sqrt{\frac{m}{3}}) = 0 \quad \square \quad m = \frac{3}{4} \quad \square \square \square \square \quad F(x) \square (0, +\infty) \quad \square \square \square \square \quad 1 \quad \square \quad x_0 = \sqrt{\frac{m}{3}} = \frac{1}{2} \quad \square \square \square \square \quad y = g(x) \quad \square \quad 3 \quad \square \square \square \square$$

$$\square \quad F(\sqrt{\frac{m}{3}}) > 0 \quad \square \quad m > \frac{3}{4} \quad \square \square \square \square \quad F(x) \square (0, +\infty) \quad \square \square \square \square \quad 2 \quad \square \square \square \square \square \square \square \quad \sqrt{\frac{m}{3}} \quad \square \square \square \square \square \quad \sqrt{\frac{m}{3}} \quad \square$$

$$\square \quad m, \frac{3}{4} \quad \square$$

$$\therefore \square \square \quad g(x) \quad \square \quad 2 \quad \square \square \quad 3 \quad \square \square \square \square$$

$$9 \square \square \square \square \square \quad f(x) = a \ln x + x - 1 \quad \square \quad g(x) = x^2 - 1 \quad \square$$

$$\square \quad 1 \square \square \square \square \quad I: y = -x + 1 \quad \square \square \square \square \quad y = f(x) \quad \square \square \square \square \square \square \square \quad a \square \square \square \square$$

$$\square \quad 2 \square \square \quad \min\{m_1, m_2\} \quad \square \square \quad m_1 \square n \square \square \square \square \square \square \square \square \square \quad h(x) = \min\{f(x), g(x)\} (x > 0) \quad \square \square \square \square \quad h(x) \quad \square \square \square \square \square \square \square \square$$

$$\square \square \square \square \square \square \square \quad 1 \square \square \square \square \square \square \quad f(x) = \frac{a}{x} + 1 \quad \square \square \square \square \quad y = f(x) \quad \square \square \quad F(x_0, y_0) \quad \square \square \square \square \square \square \square \quad y - y_0 = (\frac{a}{x_0} + 1)(x - x_0) \quad \square$$

$$\square \quad y_0 = a \ln x_0 + x_0 - 1 \quad \square \square \square \square \square \square \quad y = (\frac{a}{x_0} + 1)x + a \ln x_0 - a - 1 \quad \square \square \square \square \square \square \square \quad I \square \square \square \square \quad \begin{cases} \frac{a}{x_0} + 1 = -1 \\ a \ln x_0 - a - 1 = 1 \end{cases} \quad \square \square \square \square \quad x_0 \square \square$$

$$\frac{a}{2} \ln(-\frac{a}{2}) - \frac{a}{2} - 1 = 0 \quad \textcircled{1} \square \square \quad \Phi(x) = -x \ln x + x - 1 \quad \square \square \quad \Phi'(x) = -\ln x \quad \square \square \quad 0 < x < 1 \quad \square \quad \Phi(x) \quad \square \square \square \square \square \square \quad x > 1 \quad \square \quad \Phi(x) \quad \square \square \square \square \square \square$$

$$\therefore \Phi(x)_{\max} = \Phi \quad \square \quad 1 \quad \square = 0 \quad \square \square \textcircled{1} \quad \square \quad \Phi(-\frac{a}{2}) = 0 \quad \square \quad \therefore 1 = -\frac{a}{2} \quad \square \square \square \square \quad a = -2 \quad \square$$

$$\square \quad 2 \quad \textcircled{1} \quad \square \quad 0 < x < 1 \quad \square \square \quad g(x) = x^2 - 1 < 0 \quad \square \square \square \square \quad h(x), g(x) < 0 \quad \square \square \square \square \square \square$$

$$\textcircled{2} \quad \square \quad x = 1 \quad \square \square \quad f \quad \square \quad 1 \quad \square = g \quad \square \quad 1 \quad \square = 0 \quad \square \square \square \square \quad h \quad \square \quad 1 \quad \square = 0 \quad \square \square \quad x = 1 \quad \square \quad h(x) \quad \square \square \square \square \square \square$$

$$\textcircled{3} \quad \square \quad x > 1 \quad \square \square \quad g(x) > 0 \quad \square \square \quad h(x) \quad \square \square \square \square \square \square \quad f(x) \quad \square \square \square \square \square \square$$

$$f(x) = \frac{a}{x} + 1 = \frac{x+a}{x}$$

$$\textcircled{1} \quad a < -1 \quad f(x) > 0 \quad f(x) \text{ in } (1, +\infty) \quad f(x) > f(1) = 0 \quad h(x) \text{ in } (1, +\infty)$$

$$\textcircled{2} \quad a < -1 \quad f(x) = 0 \quad x = -a \quad f(x) \text{ in } (1, -a) \quad (-a, +\infty) \quad f(1) = 0$$

$$\therefore f(x) \text{ in } (1, -a) \quad \Phi\left(\frac{1}{x}\right), \Phi(1) = 0$$

$$\ln x, x-1 \quad x > 0 \quad \ln(4a^2) = 2\ln(-2a), 2(-2a-1)$$

$$f(4a^2) = a\ln(4a^2) + 4a^2 - 1 = a \times 2(-2a-1) + 4a^2 - 1 = -2a - 1 > 0 \quad x \in (-a, 4a^2)$$

$$x \in (-a, +\infty) \quad f(x) = 0 \quad h(x) = 0 \quad h(x) \text{ in } (1, +\infty)$$

$$a < -1 \quad h(x) \text{ in } (1, +\infty) \quad a < -1 \quad h(x) \text{ in } (2, +\infty)$$

$$f(x) = \ln(x-1) \quad g(x) = \frac{2a}{3}x^2 + 3(1-a)x^2 - 18x + 11a + 26 \quad (a < 0)$$

$$g(x) \text{ in } (1, +\infty)$$

$$\min\{m, n\} \quad m \leq n \quad f(x) = \min\{f(x), g(x)\} \quad (x > 1) \quad y = f(x) \quad a$$

$$g(x) = \frac{2a}{3}x^2 + 3(1-a)x^2 - 18x + 11a + 26 \quad R$$

$$g(x) = 2(x-3)(ax+3) \quad g(x) = 2(x-3)(ax+3)$$

$$g(x) = 0 \quad x_1 = 3, x_2 = -\frac{3}{a}$$

$$\textcircled{1} \quad -\frac{3}{a} < 3 \quad a < -1 \quad x \in (-\infty, -\frac{3}{a}) \cup (3, +\infty) \Rightarrow g(x) < 0, x \in (-\frac{3}{a}, 3) \Rightarrow g(x) > 0$$

$$\textcircled{2} \quad -\frac{3}{a} = 3 \quad a = -1 \quad g(x) = -2(x-3)^2 \leq 0$$

$$\textcircled{3} \quad -\frac{3}{a} > 3 \quad -1 < a < 0 \quad x \in (-\infty, 3) \cup (-\frac{3}{a}, +\infty) \Rightarrow g(x) < 0, x \in (3, -\frac{3}{a}) \Rightarrow g(x) > 0$$

$$a < -1 \quad g(x) \text{ in } (-\infty, -\frac{3}{a}) \cup (3, +\infty) \quad (-\frac{3}{a}, 3)$$

$$\square\square\square x>1\square\square f(x)>0\square\square\square\square$$

$$\square x>1\square\square f(x)=0\square\square\square\square$$

$$\square-1< x<1\square\square f(x)<0\square\square\square\square$$

$$\square\square f(x)\square\square\square\square\square\square\square\square g(x)\square\square\square\square$$

$$\square\square\square\square\square\square g(x)\square-1< x<1\square\square\square\square\square\square$$

$$g'(x)=2ax-1-4a\sin x+\frac{1}{x+1}\square$$

$$\square\square g'(x)=2a-4a\cos x-\frac{1}{(x+1)^2}(-1< x<1)\square\square\square$$

$$\textcircled{1}\square a>0\square\square\square\square?1< x<1\square\cos x\in(\cos 1,1]\square\square$$

$$\square y=\cos x\square\square\square(0,\frac{\pi}{2})\square\square\square\square\square\square$$

$$\square\square\cos 1>\cos\frac{\pi}{3}=\frac{1}{2}\square$$

$$\square\square-1< x<1\square\square1-2\cos x<0\square$$

$$g'(x)=2a-4a\cos x-\frac{1}{(x+1)^2}<0\square$$

$$\square\square g(x)\square\square\square\square\square\square$$

$$\square g(0)=0\square\square\square$$

$$\square -1 < x < 0 \square \square g'(x) > 0 \square g(x) \square \square \square \square$$

$$\square 0 < x < 1 \square \square g'(x) < 0 \square g(x) \square \square \square \square$$

$$\square x \rightarrow ?_1 \square \square \ln(x+1) \rightarrow ?_x \square$$

$$\square \square g(x) \rightarrow ?_{\infty} \square$$

$$\square x=0 \square \square g(0)=4a>0 \square$$

$$\square g_{\square 1 \square} = a \cdot 1 + 4a \cos 1 + \ln 2 \square f_{\square 1 \square} = 0 \square$$

$$\square g_{\square 1 \square} > 0 \Rightarrow a > \frac{1 - \ln 2}{1 + 4 \cos 1} \square \square \square F(x) \square 1 \square \square \square$$

$$\square g(1) > 0 \Rightarrow a > \frac{1 - \ln 2}{1 + 4 \cos 1} \square \square \square F(x) \square 2 \square \square \square$$

$$\square g_{\square 1 \square} < 0 \Rightarrow 0 < a < \frac{1 - \ln 2}{1 + 4 \cos 1} \square \square \square F(x) \square 3 \square \square \square$$

$$\textcircled{2} \square a=0 \square \square g(x) = \ln(x+1) \cdot x \square$$

$$\square \textcircled{1} \square -1 < x < 0 \square \square g'(x) > 0 \square g(x) \square \square \square \square$$

$$\square 0 < x < 1 \square \square g'(x) < 0 \square g(x) \square \square \square \square$$

$$\square \square g(x)_{\max} = g(0) = 0 \square$$

$$g_{\square 1 \square} = \ln 2 - 1 < 0 \square$$

$$\text{Se } a=0 \text{ temos } F(x)$$

$$\textcircled{3} \text{ Se } a < 0 \text{ temos } g(x) = a(x^2 + 4\cos x) - x + \ln(x+1)$$

$$a(x^2 + 4\cos x) < 0$$

$$- x + \ln(x+1), 0$$

$$g(x) < 0$$

$$f'(1) = 0$$

$$\text{Se } a < 0 \text{ temos } F(x) \text{ é 1 vez}$$

$$\text{Se } a > \frac{1 - \ln 2}{1 + 4\cos 1} \text{ e } a < 0 \text{ temos } F(x) \text{ é 1 vez}$$

$$\text{Se } a = \frac{1 - \ln 2}{1 + 4\cos 1} \text{ e } a = 0 \text{ temos } F(x) \text{ é 2 vezes}$$

$$\text{Se } 0 < a < \frac{1 - \ln 2}{1 + 4\cos 1} \text{ temos } F(x) \text{ é 3 vezes}$$

$$13 \text{ Se } f(x) = (x-2)e^{x-1} - \frac{1}{2}x^2 + x + \frac{1}{2} \text{ e } g(x) = ax^2 - x + 4a\cos x + \ln(x+1) \text{ com } a \in \mathbb{R}$$

$$1 \text{ Se } f(x) \text{ é uma função estritamente crescente } f(x) > 0$$

$$2 \text{ Se } \max\{m, n\} \leq m \leq n \text{ temos } F(x) = \max\{f(x), g(x)\} \text{ é uma função}$$

$$\text{Se } f(x) = (x-1)e^{x-1} - x + 1 = (x-1)(e^{x-1} - 1)$$

$$\square x > 1 \square \square x - 1 > 0 \square \square e^{x-1} - 1 > 0 \square \square f'(x) > 0 \square$$

$$\square x < 1 \square \square x - 1 < 0 \square \square e^{x-1} - 1 < 0 \square \square f'(x) > 0 \square$$

$$\square x = 1 \square \square f' \square \square = 0 \square$$

$$\square \square \square x \in R \square \square f(x) \dots 0 \square \square f(x) \square \square R \square \square \square \square \square \square$$

$$\square f' \square \square = 0 \square \square \square f(x) > 0 \square \square \square (1, +\infty) \square$$

$$\square 2 \square \square \square F(x) \square \square \square \square (-1, +\infty) \square$$

$$\square 1 \square \square \square \square f(x) \square \square R \square \square \square \square \square f' \square \square = 0 \square$$

$$\square x > 1 \square \square f(x) > 0 \square \square F(x) = \max\{f(x) \square g(x)\} \square$$

$$\square \square \square x > 1 \square \square F(x) > 0 \square \square \square \square \square x > 1 \square \square F(x) = 0 \square \square \square$$

$$\square -1 < x < 1 \square \square f(x) < 0 \square \square \square$$

$$\square \square F(x) \square \square \square \square \square \square g(x) \square \square \square$$

$$\square \square \square \square \square g(x) \square \square -1 < x < 1 \square \square \square \square \square$$

$$g(x) = 2ax - 1 - 4a \sin x + \frac{1}{x+1} \square$$

$$\square \square g'(x) = 2a - 4a \cos x - \frac{1}{(x+1)^2} (-1 < x < 1) \square \square$$

$$\textcircled{1} \square a > 0 \square \square \square \square -1 < x < 1 \square \square \cos x \in (\cos 1, 1) \square$$

$$\square \square y = \cos x \square \square \square \left(0, \frac{\pi}{2}\right) \square \square \square \square$$

$$\square \square \cos 1 > \cos \frac{\pi}{3} = \frac{1}{2} \square$$

$$\square -1 < x < 1 \square \square 1 - 2 \cos x < 0 \square \mathcal{G}'(x) = 2a(1 - \cos x) - \frac{1}{(x+1)^2} < 0 \square$$

$$\square \mathcal{G}'(x) \square \square \square \square \square \mathcal{G}'(0) = 0 \square \square$$

$$\square -1 < x < 0 \square \square \mathcal{G}'(x) > 0 \square \mathcal{G}(x) \square \square \square \square \square$$

$$\square 0 < x < 1 \square \square \mathcal{G}'(x) < 0 \square \mathcal{G}(x) \square \square \square \square \square$$

$$\square x \rightarrow -1 \square \square \ln(x+1) \rightarrow -x \square$$

$$\square \mathcal{G}(x) \rightarrow -\infty \square$$

$$\square x=0 \square \square \mathcal{G}(0) = 4a > 0 \square$$

$$\square \mathcal{G}'(1) = a - 1 + 4a \cos 1 + \ln 2 \square \square \mathcal{F}'(1) = 0 \square$$

$$\square \mathcal{G}'(1) > 0 \Rightarrow a > \frac{1 - \ln 2}{1 + 4 \cos 1} \square \square \square \mathcal{F}(x) \square 1 \square \square \square \square$$

$$\square \mathcal{G}'(1) = 0 \Rightarrow a = \frac{1 - \ln 2}{1 + 4 \cos 1} \square \square \square \mathcal{F}(x) \square 2 \square \square \square \square$$

$$\square \mathcal{G}'(1) < 0 \Rightarrow 0 < a < \frac{1 - \ln 2}{1 + 4 \cos 1} \square \square \square \mathcal{F}(x) \square 3 \square \square \square \square$$

$$\textcircled{2} \square a=0 \square \square \mathcal{G}(x) = \ln(x+1) - x \square$$

$$\square \textcircled{1} \square -1 < x < 0 \square \square \mathcal{G}'(x) > 0 \square \mathcal{G}(x) \square \square \square \square \square$$

$$\square 0 < x < 1 \square \square \mathcal{G}'(x) < 0 \square \mathcal{G}(x) \square \square \square \square \square$$

$$\square \mathcal{G}(x)_{\max} = \mathcal{G}(0) = 0 \square \square \mathcal{G}'(1) = \ln 2 - 1 < 0 \square$$

$$\square \square a=0 \square \square \square \mathcal{F}(x) \square \square \square \square \square \square$$

$$\textcircled{3} \square a < 0 \square \square \mathcal{G}(x) = a(x^2 + 4 \cos x) - x + \ln(x+1) \square$$

$$a(x^2 + 4\cos x) < 0 \quad -x + \ln(x+1), 0 \quad g(x) < 0 \quad f(1) = 0$$

$$a < 0 \quad F(x) \quad 1$$

$$a > \frac{1 - \ln 2}{1 + 4\cos 1} \quad a < 0 \quad F \quad 1 \quad 1$$

$$a = \frac{1 - \ln 2}{1 + 4\cos 1} \quad a = 0 \quad F(x) \quad 2$$

$$0 < a < \frac{1 - \ln 2}{1 + 4\cos 1} \quad F(x) \quad 3$$

$$14 \quad f(x) = e^x - 2ax - a \quad g(x) = \ln x$$

$$1 \quad f(x)$$

$$2 \quad \max\{m, n\} \quad m, n \quad h(x) = \max\{f(x), g(x)\} (x > 0) \quad a$$

$$1 \quad f(x) \quad R \quad f(x) = e^x - 2a$$

$$a, 0 \quad f(x) > 0 \quad x \in R \quad f(x) \quad R$$

$$a > 0 \quad f(x) = 0 \quad x = \ln(2a)$$

$$x \in (-\infty, \ln(2a)) \quad f(x) < 0 \quad x \in (\ln(2a), +\infty) \quad f(x) > 0$$

$$f(x) \quad (-\infty, \ln(2a)) \quad (\ln(2a), +\infty)$$

$$2 \quad x \in (1, +\infty) \quad g(x) = \ln x > 0$$

$$h(x) = \max\{f(x), g(x)\} \quad g(x) > 0$$

$$h(x) \quad (1, +\infty)$$

$$2 \quad x = 1 \quad f(1) = e - 3a$$

$$a, \frac{e}{3} \quad h(1) = \max\{f(1), g(1)\} = g(1) = 0 \quad x = 1 \quad h(x)$$

$$\square \quad a < \frac{e}{3} \quad \square \quad h(1) = \max\{f(1), g(1)\} = f(1) > 0 \quad \square \quad x=1 \quad \square \quad h(x) \quad \square \quad \square \quad \square$$

$$\textcircled{3} \quad \square \quad x \in (0,1) \quad \square \quad g(x) = hx < 0 \quad \square$$

$$\square \quad h(x) \quad \square \quad (0,1) \quad \square \quad \square \quad \square \quad \square \quad \square \quad \square \quad f(x) \quad \square \quad (0,1) \quad \square \quad \square \quad \square \quad \square \quad \square$$

$$f(x) \quad \square \quad (0,1) \quad \square \quad \square \quad \square \quad \square \quad \square \quad f(x) = 0 \quad \square \quad (0,1) \quad \square \quad \square \quad \square \quad \square \quad \square \quad \frac{e^x}{2x+1} = a \quad \square \quad (0,1) \quad \square \quad \square \quad \square \quad \square \quad \square$$

$$\square \quad \square \quad \varphi(x) = \frac{e^x}{2x+1} \quad \square \quad x \in (0,1) \quad \square$$

$$\square \quad \varphi'(x) = \frac{(2x-1)e^x}{(2x+1)^2} \quad \square$$

$$\square \quad \varphi(x) \quad \square \quad (0, \frac{1}{2}) \quad \square \quad \square \quad \square \quad \square \quad \square \quad (\frac{1}{2}, 1) \quad \square \quad \square \quad \square \quad \square$$

$$\square \quad \varphi(0) = 1 \quad \square \quad \varphi(1) = \frac{e}{3} \quad \square \quad \varphi(\frac{1}{2}) = \frac{\sqrt{e}}{2} \quad \square$$

$$\square \quad a < \frac{\sqrt{e}}{2} \quad \square \quad a.1 \quad \square \quad f(x) \quad \square \quad (0,1) \quad \square \quad \square \quad \square \quad \square$$

$$\square \quad a = \frac{\sqrt{e}}{2} \quad \square \quad \frac{e}{3}, \quad a < 1 \quad \square \quad f(x) \quad \square \quad (0,1) \quad \square \quad \square \quad \square \quad \square \quad \square$$

$$\square \quad \frac{\sqrt{e}}{2} < a < \frac{e}{3} \quad \square \quad f(x) \quad \square \quad (0,1) \quad \square \quad \square \quad \square \quad \square \quad \square$$

$$\square \quad \square \quad \square \quad a = \frac{\sqrt{e}}{2} \quad \square \quad h(x) \quad \square \quad (0, +\infty) \quad \square \quad \square \quad 1 \quad \square \quad \square \quad \square$$

$$\square \quad \frac{\sqrt{e}}{2} < a < 1 \quad \square \quad h(x) \quad \square \quad (0, +\infty) \quad \square \quad \square \quad \square \quad \square \quad \square$$

$$\square \quad a.1 \quad \square \quad h(x) \quad \square \quad (0, +\infty) \quad \square \quad \square \quad 1 \quad \square \quad \square \quad \square$$

$$H(x) = \begin{cases} f(x), & 0 < x < 1 \\ g(x), & x > 1 \end{cases} \quad \text{where } f(x) = 4\ln x + \frac{2x+1}{x^2} + a - 3, \quad g(x) = 4\ln x$$

$$f(x) = 4\ln x + \frac{2x+1}{x^2} + a - 3, \quad g(x) = 4\ln x$$

$$f(x) = \left(\frac{1}{x} - 1\right)^2 + a$$

$$H(x) = \max\{f(x), g(x)\}$$

$$f(x) = 4\ln x + \frac{2x+1}{x^2} + a - 3, \quad g(x) = 4\ln x + \frac{1}{x} - 1$$

$$f'(x) = 4\left(\frac{1}{x} - \frac{1}{x^2}\right) = \frac{4(x-1)}{x^2}$$

$$0 < x < 1 \implies f'(x) < 0, \quad x > 1 \implies f'(x) > 0$$

$$f(x) \text{ is decreasing on } (0, 1) \text{ and increasing on } (1, +\infty)$$

$$f(1) = g(1) = 0$$

$$f(x) = \left(\frac{1}{x} - 1\right)^2 + a$$

$$f(x) = \frac{4}{x} - \frac{2}{x^2} - \frac{2}{x^2} = \frac{2(2x+1)(x-1)}{x^2}$$

$$0 < x < 1 \implies f(x) < 0, \quad x > 1 \implies f(x) > 0$$

$$f(x) \text{ is decreasing on } (0, 1) \text{ and increasing on } (1, +\infty)$$

$$f(1) = g(1) = a$$

$$f(x) - g(x) = \frac{2x+1}{x^2} - 3 = \frac{(x-1)(3x+1)}{x^2}$$

$$0 < x < 1 \implies f(x) > g(x), \quad x = 1 \implies f(x) = g(x), \quad x > 1 \implies f(x) < g(x)$$

$$H(x) = \begin{cases} f(x), & 0 < x < 1 \\ g(x), & x > 1 \end{cases}$$

$$(ii) \quad a > 0 \quad f(x) - g(x) = -\frac{(x-1)(3x+1)}{x^2} + a$$

$$0 < x < 1 \quad f(x) > g(x) \quad h(x) = f(x) \quad a > 0$$

$$x > 1 \quad f(x) > a > 0 \quad g(x) > 0 \quad h(x) > 0$$

$$h(x)$$

$$(iii) \quad a < 0 \quad f(x) \cdot \left(\frac{1}{x} - 1\right)^2 + a \quad 0 < \frac{1}{\sqrt{-a}+1} < 1 \quad f\left(\frac{1}{\sqrt{-a}+1}\right) > (2\sqrt{-a}+1-1)^2 + a = 0$$

$$f(x)_{min} = f(1) = a < 0 \quad f(x) \quad (0,1) \quad x$$

$$c \in (x_0, 1) \quad f(c) = g(c) \quad \frac{2c+1}{c^2} + a - 3 = 0 \quad 3 - a = \frac{2c+1}{c^2}$$

$$x > c \quad g(x) - f(x) = -\frac{2x+1}{x^2} + \frac{2c+1}{c^2} - a + 3 = \frac{x-c}{cx} \left(\frac{c+x}{cx} + 2\right) > 0$$

$$g(x) > f(x) \quad h(x) = \begin{cases} f(x), & 0 < x < c \\ g(x), & x > c \end{cases}$$

$$h(x) \quad x_0 \quad 1 \quad a < 0 \quad h(x)$$

$$a = 0 \quad h(x) \quad a > 0 \quad h(x) \quad a < 0 \quad h(x)$$

$$16 \quad f(x) = \ln x - ax + a \quad g(x) = x^2 - 1$$

$$1 \quad a = 0 \quad x > 0 \quad x \neq 1 \quad \frac{1+x}{1-x} f(x) < \frac{2}{1-x^2} g(x)$$

$$2 \quad m \neq n \quad m, n \in \mathbb{N} \quad h(x) = \max\{f(x), g(x)\} \quad (x > 0) \quad h(x)$$

$$1 \quad a = 0 \quad f(x) = \ln x$$

$$\frac{1+x}{1-x} f(x) < \frac{2}{1-x^2} g(x) \quad \frac{1}{1-x} [(1+x) \ln x - 2(x-1)] < 0 \quad \frac{1}{1-x} \left[\ln x - \frac{2(x-1)}{x+1} \right] < 0$$

$$\square\square\square\square x>1\square\square \ln x > \frac{2(x-1)}{x+1} \square\square 0 < x < 1\square\square \ln x < \frac{2(x-1)}{x+1} \square$$

$$\square \varphi(x) = \ln x - \frac{2(x-1)}{x+1} \square\square \varphi'(x) = \frac{1}{x} - \frac{4}{(x+1)^2} = \frac{(x-1)^2}{x(x+1)^2} > 0 \square$$

$$\therefore \varphi(x) \square (0,1) \square\square\square\square\square\square\square (1,+\infty) \square\square\square\square\square\square$$

$$\therefore \square 0 < x < 1\square\square \varphi(x) < \varphi \square 1\square = 0 \square\square\square \frac{1}{1-x} [\ln x - \frac{2(x-1)}{x+1}] < 0 \square$$

$$\square x > 1\square\square \varphi(x) > \varphi \square 1\square = 0 \square\square\square \frac{1}{1-x} [\ln x - \frac{2(x-1)}{x+1}] < 0 \square$$

$$\square a = 0\square x > 0\square x \neq 1\square\square \frac{1+x}{1-x} f(x) < \frac{2}{1-x^2} g(x) \square$$

$$\square 2\square\square\square (i) \square x > 1\square\square g(x) > 0 \square h(x) \dots g(x) > 0 \square \therefore h(x) \square (1,+\infty) \square\square\square\square\square$$

$$(ii) \square x = 1\square\square g \square 1\square = f \square 1\square = 0 \square\square h \square 1\square = 0 \square \therefore x = 1 \square h(x) \square\square\square\square\square\square$$

$$(iii) \square 0 < x < 1\square\square g(x) < 0 \square \therefore g(x) \square (0,1) \square\square\square\square\square$$

$$\therefore h(x) \square (0,1) \square\square\square\square\square\square\square\square f(x) \square (0,1) \square\square\square\square\square\square\square$$

$$\square f(x) = \frac{1}{x} - a (0 < x < 1) \square$$

$$\therefore \textcircled{1} \square a, 1\square\square f(x) = \frac{1}{x} - a > 0 \square \therefore f(x) \square (0,1) \square\square\square\square\square\square f(x) < f \square 1\square = 0 \square\square\square f(x) \square\square\square\square$$

$$\textcircled{2} \square a > 1\square \square 0 < \frac{1}{a} < 1 \square\square\square f(x) > 0 \square\square 0 < x < \frac{1}{a} \square\square f(x) < 0 \square\square \frac{1}{a} < x < 1 \square \therefore f(x) \square (0, \frac{1}{a}) \square\square\square\square\square\square\square (\frac{1}{a}, 1) \square\square\square\square\square\square$$

$$\therefore f(x)_{\max} = f(\frac{1}{a}) = a - 1 - \ln a \square$$

$$\square t \square a\square = a - 1 - \ln a (a > 1) \square\square t \square a\square = 1 - \frac{1}{a} > 0 \square t \square a\square\square (1, +\infty) \square\square\square\square\square\square$$

$$\therefore t \square a\square > t \square 1\square = 0 \square\square f(\frac{1}{a}) = a - 1 - \ln a > 0 \square\square a - 1 > \ln a \square$$

$$\square\square\square\square\square\square e^{e^{-1}} > e^{e^{\ln a}} \square\square e^e > ae > a \square$$

$$\therefore 0 < e^{-a} < \frac{1}{a}$$

$$\square \quad f(e^{-a}) = -a + a(1 - e^{-a}) = -ae^{-a} < 0 \quad \square$$

□□□□□□□□□□ $f(x)$ □ $(0,1)$ □□□□□□□□ x_0 □□ $x_0 \in (e^{-a}, \frac{1}{a})$ □
 □□□□□

$$\square \quad a, 1 \square \square \quad h(x) \square \square \square \square \square \square$$

$$\square \quad a > 1 \square \square \quad h(x) \square \square \square \square \square \square$$

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